Imperial College London

7 – Morphological Operations Part 2

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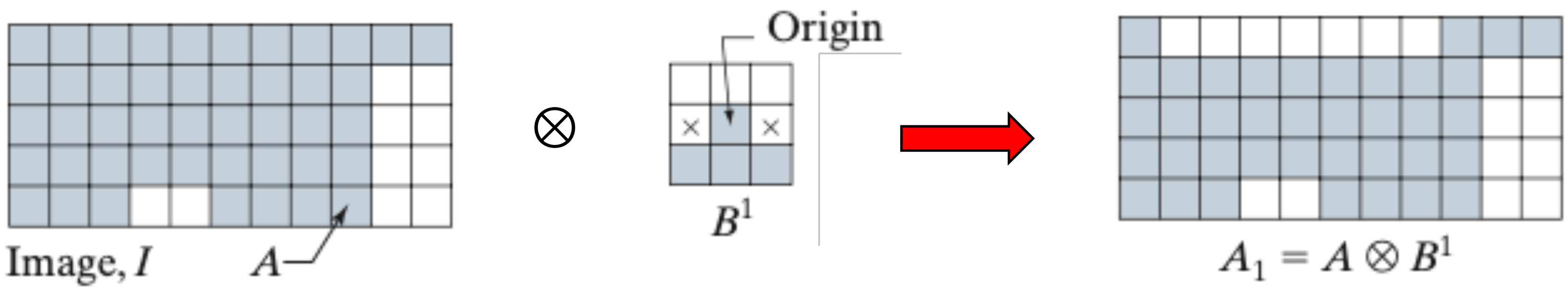
Thinning Operation (1)

- Thinning operation of a set A of foreground pixel with a structuring element B is defined as:.

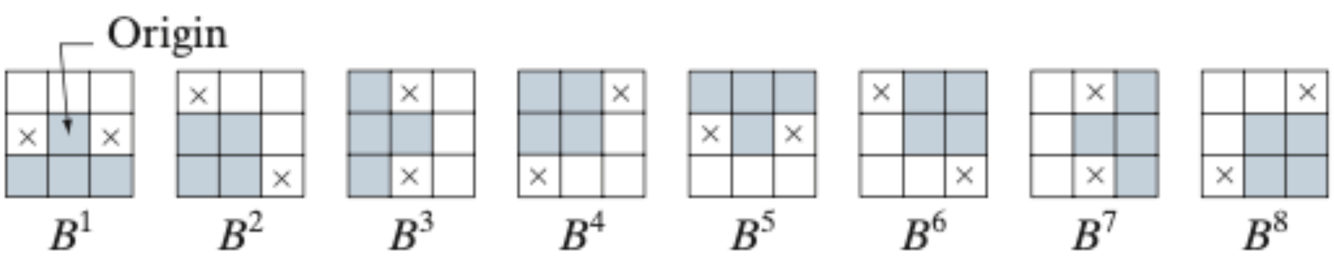
$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

$\circledast$ = Hit-or-Miss operation



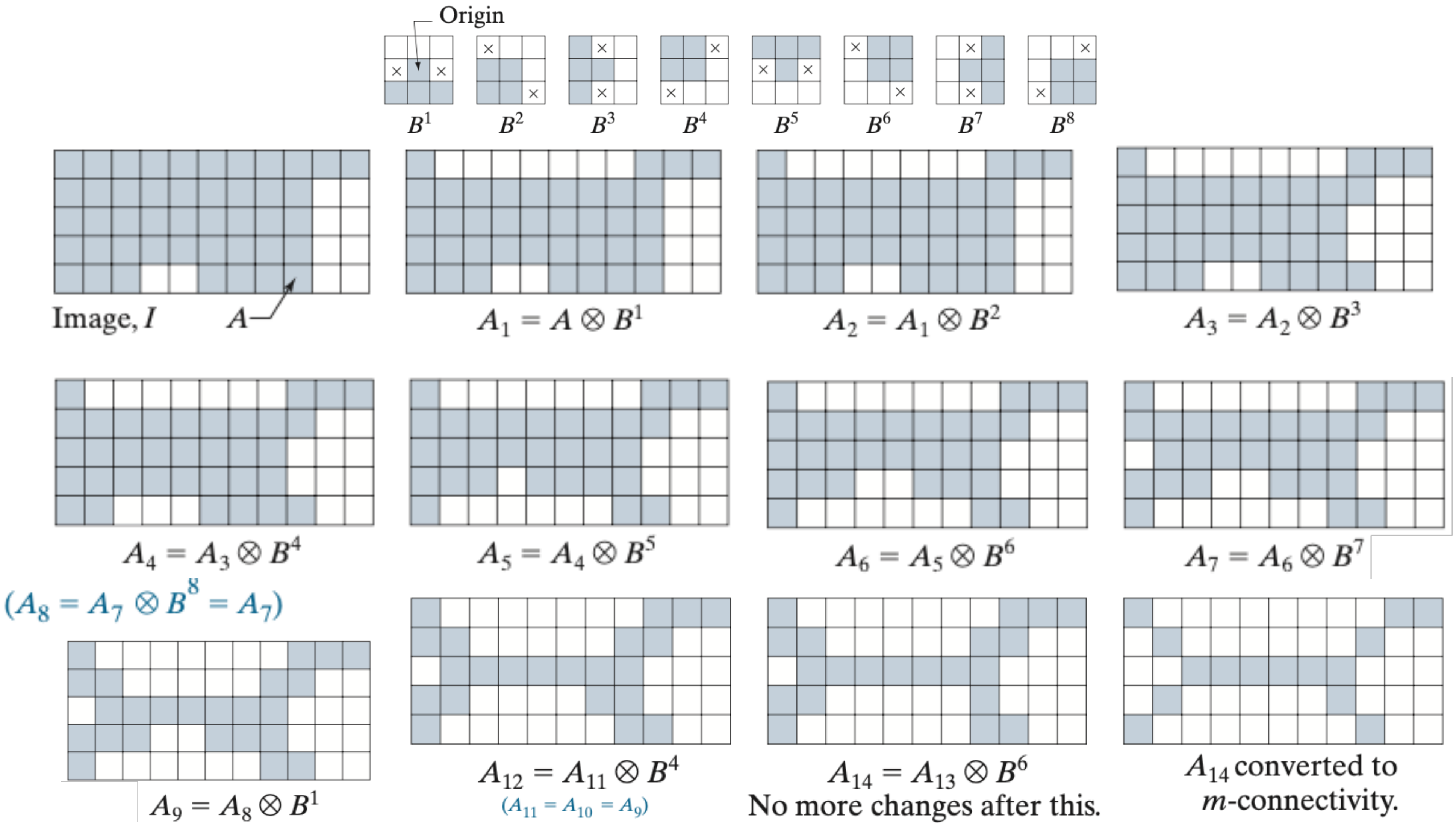
- More useful is to repeatedly apply a sequence of structuring elements to A, where

$$\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$$



x = don't care

Thinning Operation (2)



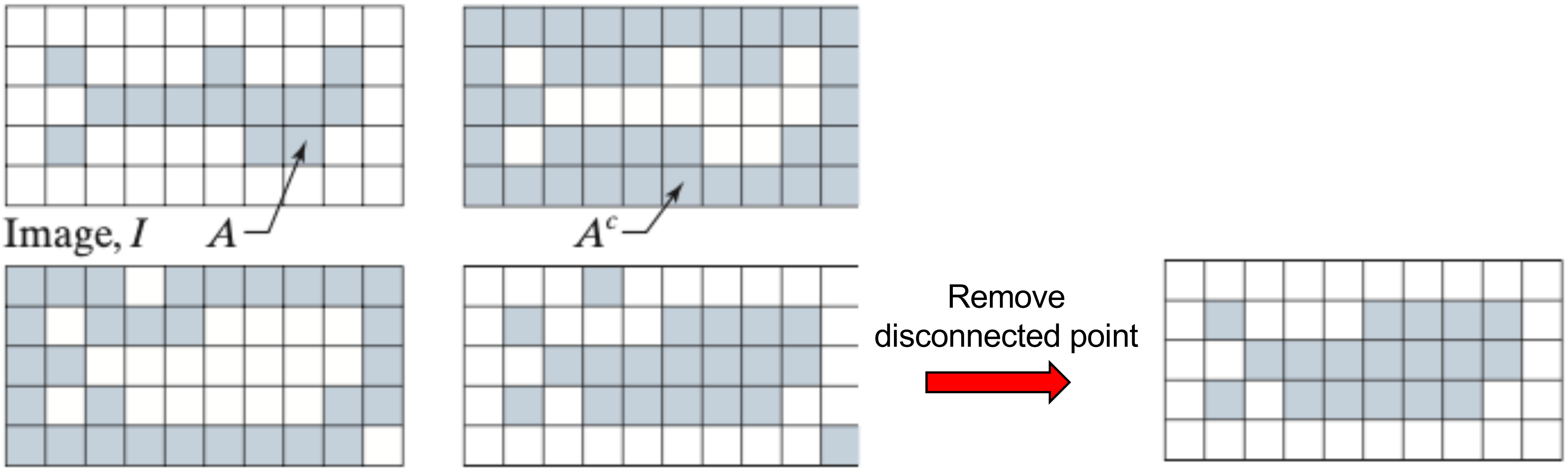
Thickening Operation

- Thickening is the morphological dual of thinning and is defined by:

$$A \odot B = A \cup (A \circledast B)$$

$\circledast$ = Hit-or-Miss operation

- We can achieve thickening by:
  1. Obtaining A^c the complement of A.
  2. Applying thinning procedure as stated in the previous slides.
  3. Take the complement of that result.



Morphological Reconstruction

- Morphological reconstruction has two basic operations: **geodesic dilation** and **erosion**, which involves two images: the **marker** and the **mask**.
- F denote the **marker** and G the **mask**, both are binary and $F \subseteq G$.
- The **geodesic dilation** of size 1 of the marker F with respect to the mask G is:

$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

- The geodesic dilation of size 2 of F with respect to G is:

$$D_G^{(2)}(F) = D_G^{(1)}\,(D_G^{(1)}(F))$$

- This can be generalized to a recursive relationship as:

$D_G^{(n)}(F) = D_G^{(1)}\,(D_G^{(n-1)}(F))$, where $n \geq 1$ and $D_G^{(0)}(F) = F$.

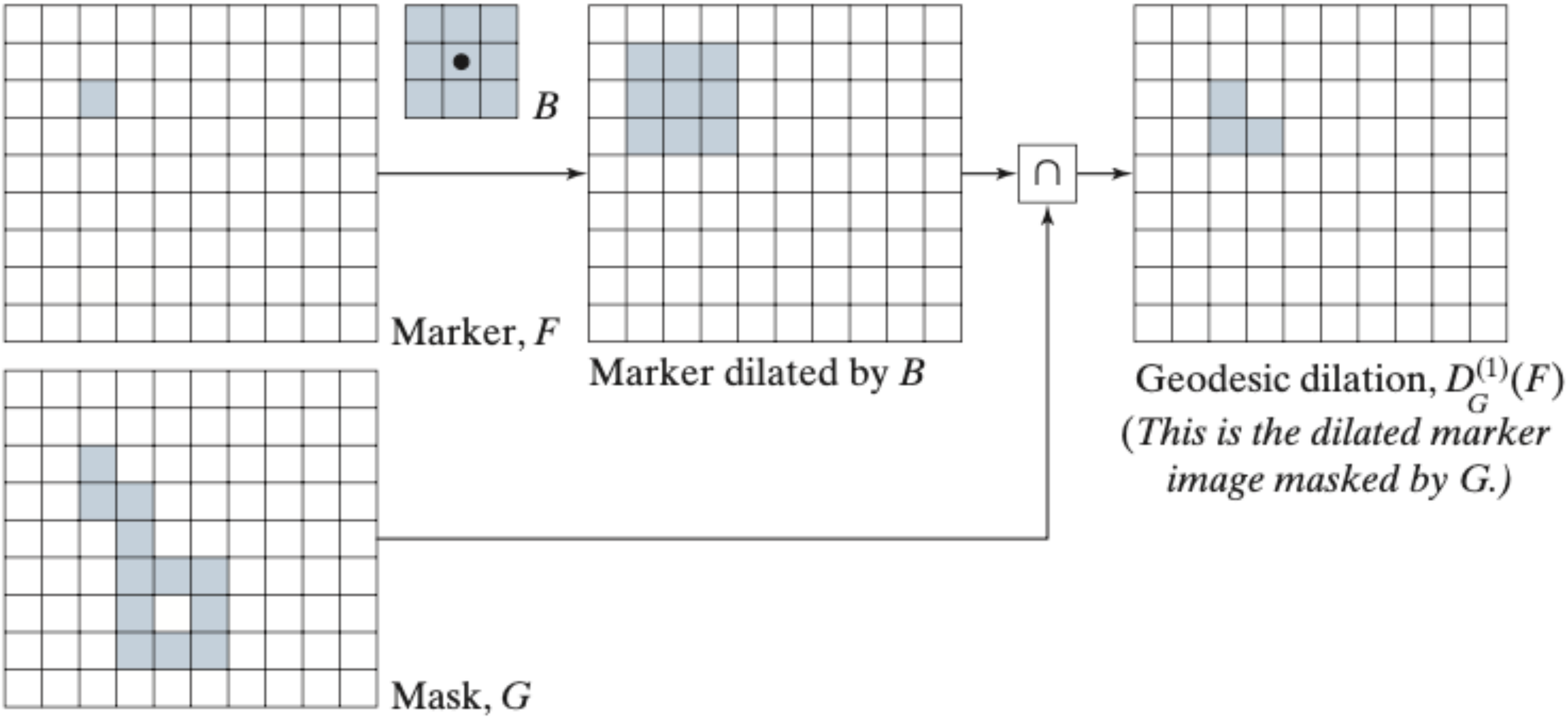
- Similarly, geodesic erosion of size *1* is defined by:

$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

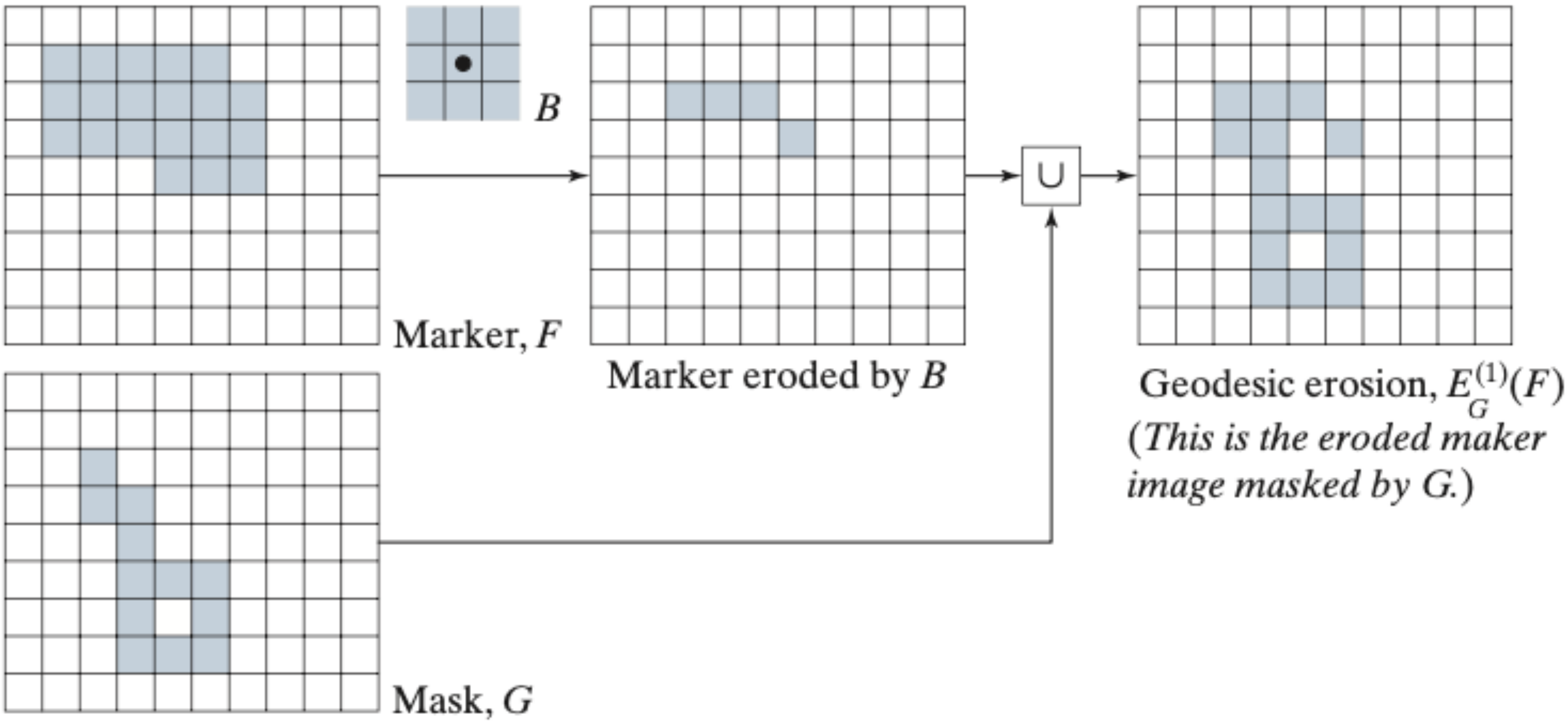
- In general:

$$E_G^{(n)}(F) = E_G^{(1)}(E_G^{(n-1)}(F))$$

Geodesic Dilation



Geodesic Erosion

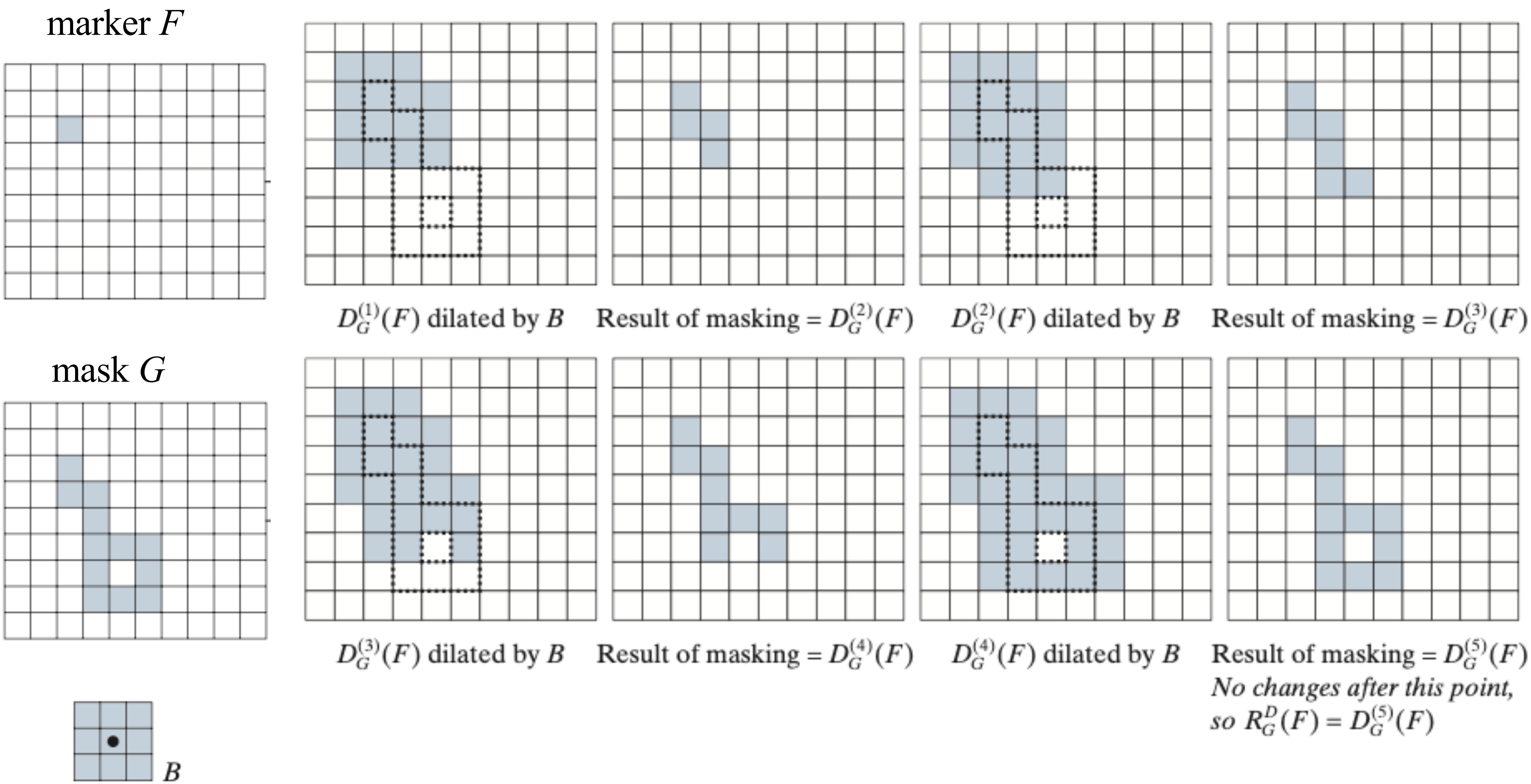


Morphological Reconstruction by Dilation

- Morphological reconstruction by dilation of a **marker** F with respect to a mask G is simply defined as the geodesic dilation of F, iterated until there are no change (i.e. achieved stability).
- Mathematically, it is formulated as:

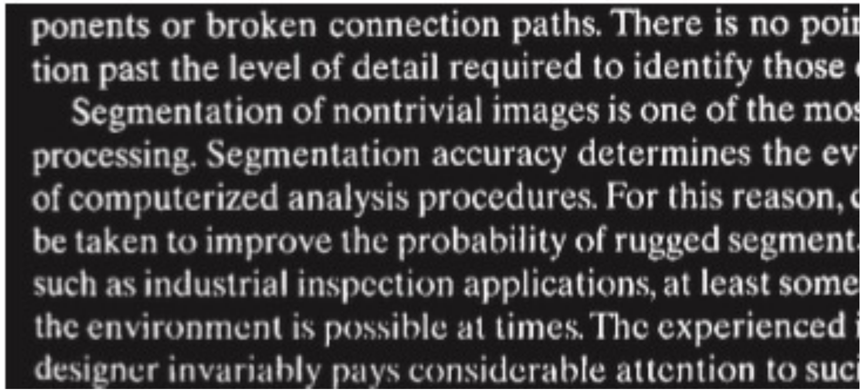
$$R_G^D(F) = D_G^{(k)}(F), \text{ for } k \text{ iterations until } D_G^{(k)}(F) = D_G^{(k-1)}(F).$$

Demonstrate how MR by dilation works



Example application of MR by dilation

G is original image



ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

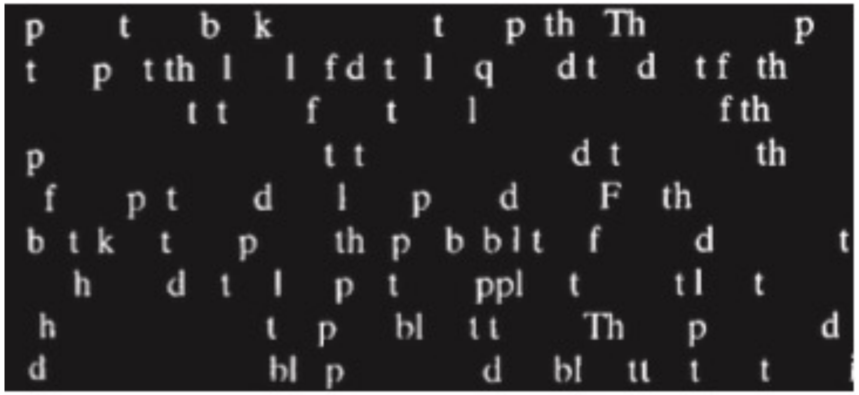
erode with B = 51x1 1's

G as mark

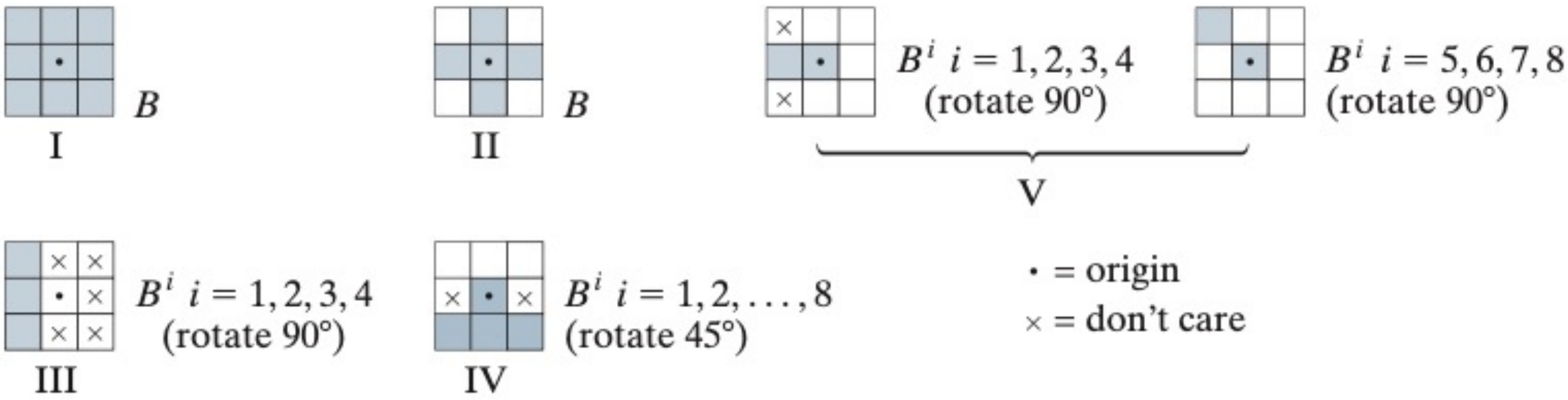
marker

MR by repeated GD

dilation with B
(erosion then dilation = opening



Five basic types of structuring elements in Binary Morphology



The Roman numerals in the third column of the table in the next few slides refer to the structuring elements used in the operation.

Summary of Binary Morphological Operators (1)

Operation	Equation	Comments
Translation	$(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$	Reflects B about its origin.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points in A, but not in B.
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	Erodes the boundary of A. (I)
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	Dilates the boundary of A. (I)

Summary of Binary Morphological Operators (2)

Operation	Equation	Comments
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$I \circledast B = \{z \mid (B)_z \subseteq I\}$	Finds instances of B in image I. B contains *both* foreground and background elements.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap I^c$ $k = 1, 2, 3, \ldots$	Fills holes in A. X_0 is of same size as I, with a 1 in each hole and 0's elsewhere. (II)

Summary of Binary Morphological Operators (3)

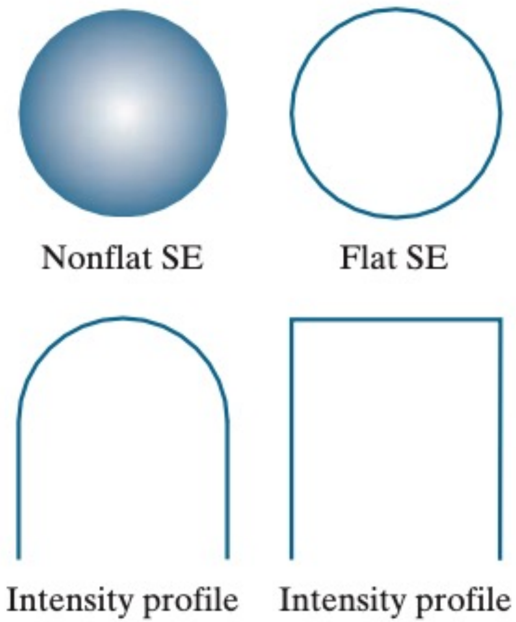
Operation	Equation	Comments
Connected components	$X_k = (X_{k-1} \oplus B) \cap I \quad k = 1, 2, 3, \ldots$	Finds connected components in I. X_0 is a set, the same size as I, with a 1 in each connected component and 0's elsewhere. (I)
Thinning	$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$ $A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$	Thickens set A using a sequence of structuring elements, as above. Uses (IV) with 0's and 1's reversed.

Summary of Binary Morphological Operators (4)

Operation	Equation	Comments
Geodesic dilation–size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and the *mask* images, respectively. (I)
Geodesic dilation–size n	$D_G^{(n)}(F) = D_G^{(1)}\left(D_G^{(n-1)}(F)\right)$	Same comment as above.
Geodesic erosion–size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	Same comment as above.
Geodesic erosion–size n	$E_G^{(n)}(F) = E_G^{(1)}\left(E_G^{(n-1)}(F)\right)$	Same comment as above.
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	With k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$.

Grayscale Morphology

- Let us apply basic operation of dilation, erosion, opening and closing tp grayscale instead of just binary images.
- For this discussion, $f(x, y)$ is a grayscale image and $b(x, y)$ is a structuring element. These are functions that assign an intensity value to each distinct pair of coordinate (x, y).
- The structuring elements may take various forms:

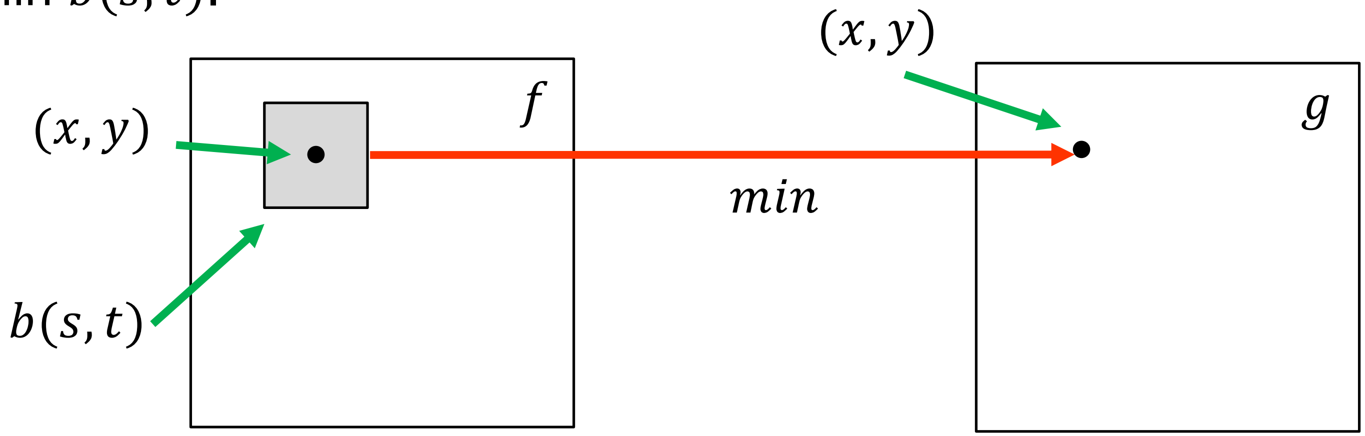


Grayscale Erosion

- **Erosion** of $f(x, y)$ by the **flat** structuring element $b(x, y)$ is defined by:

$$[f \ominus b]\,(x, y) = \min_{(s,t)\in b} \{f(x + s, y + t)\}$$

- This implies that the output intensity $g(x, y)$ is found by the **minimum** values within $b(s, t)$.



- **Erosion** of $f(x, y)$ by the **nonflat** structuring element $b(x, y)$ is defined by:

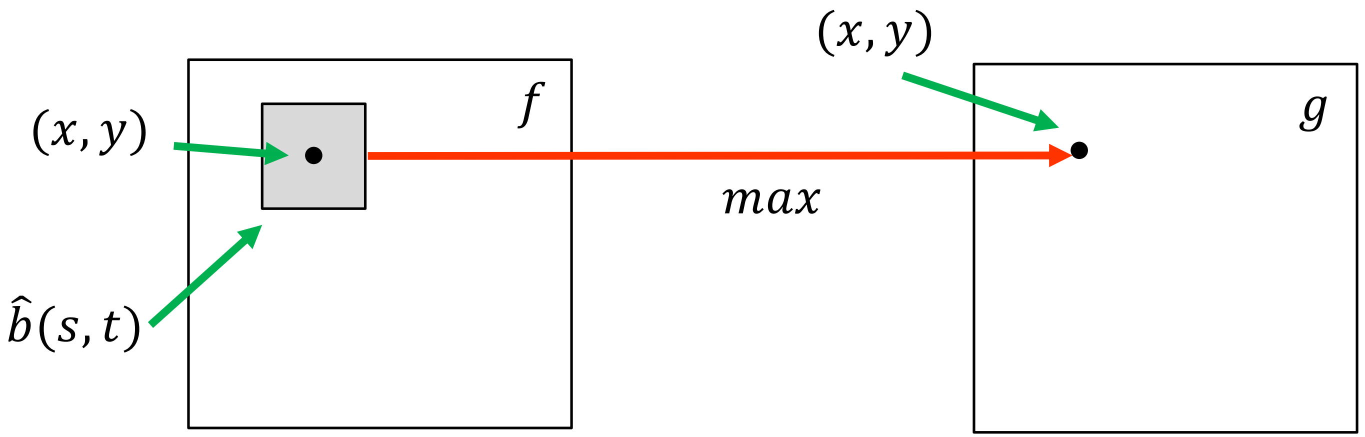
$$[f \ominus b_N]\,(x, y) = \min_{(s,t)\in b_N} \{f(x + s, y + t) - b_N(s, t)\}$$

Grayscale Dilation

- **Dilation** of $f(x,y)$ by the **flat** structuring element $b(x,y)$ is defined by:

$$[f \oplus b]\,(x,y) = \max_{(s,t)\in\hat{b}} \{f(x-s, y-t)\}$$

- This implies that the output intensity $g(x,y)$ is found by the **maximum** values within $\hat{b}(s,t)$ which is $b(s,t)$ reflect at the origin.



- **Dilation** of $f(x,y)$ by the **nonflat** structuring element $b(x,y)$ is defined by:

$$[f \oplus b_N]\,(x,y) = \max_{(s,t)\in\hat{b}_N} \left\{f(x-s, y-t) + \hat{b}_N\,(s,t)\right\}$$

Grayscale Opening and Closing

- **Opening** of $f(x, y)$ by the structuring element $b(x, y)$ is defined in a similar way to for binary images:

$$f \circ b = (f \ominus b) \oplus b$$

- For **Closing**:

$$f \cdot b = (f \oplus b) \ominus b$$

- Opening and closing for grayscale images are duals with respect to complementation and SE reflection:

$$(f \cdot b)^c = f^c \circ \hat{b}$$

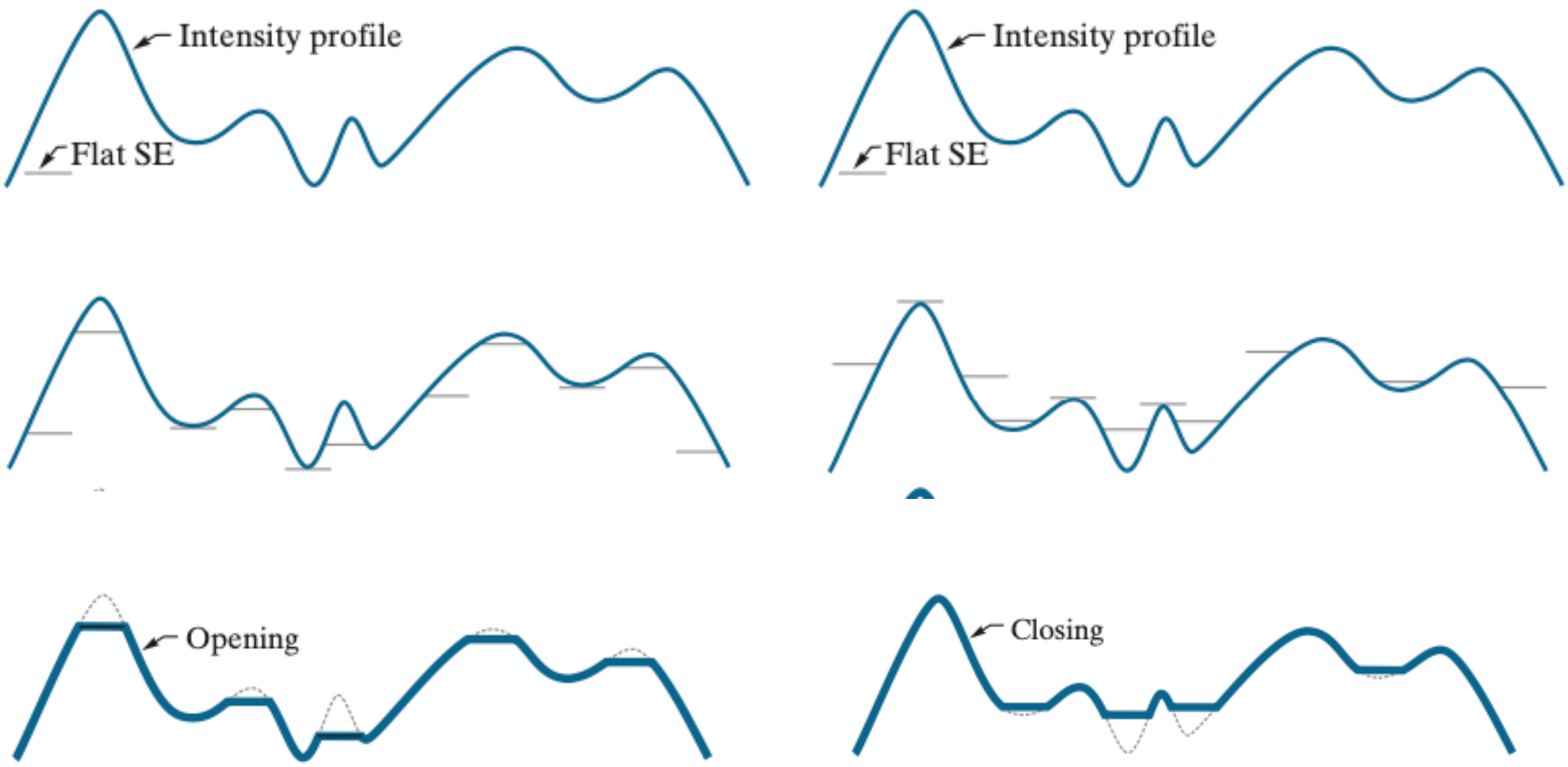
and

$$(f \circ b)^c = f^c \cdot \hat{b}$$

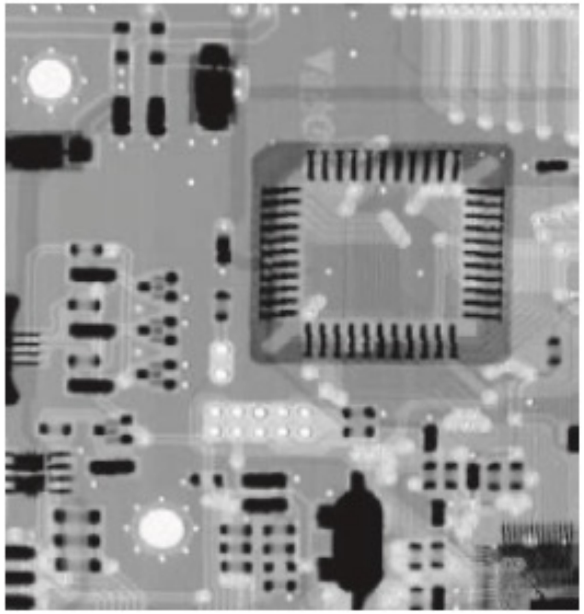
- Because $f^c = -f$,

$$(f \cdot b)^c = -(f \cdot b) = (-f \circ b)$$

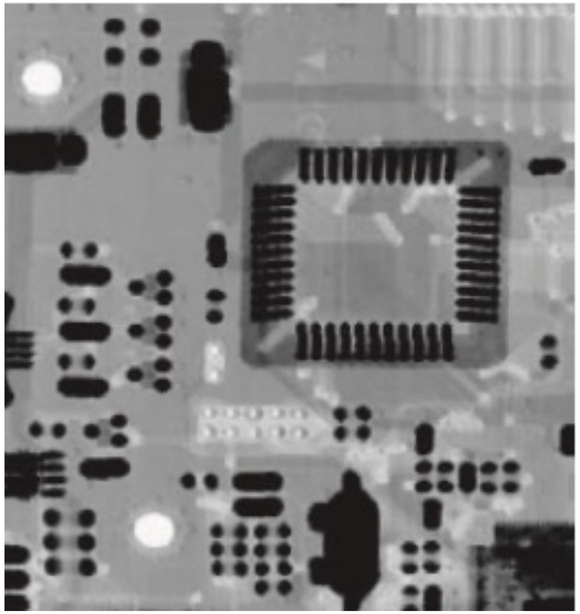
Grayscale Opening and Closing in 1D



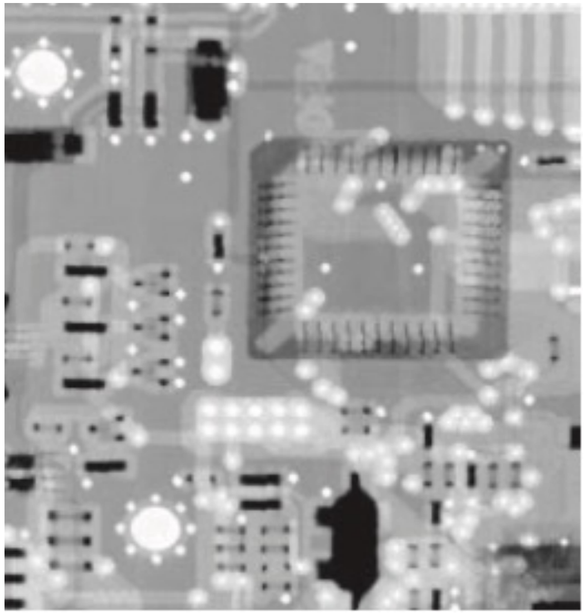
Example of Grayscale – Erosion and Dilation



448 x 425
X-ray image of a PCB

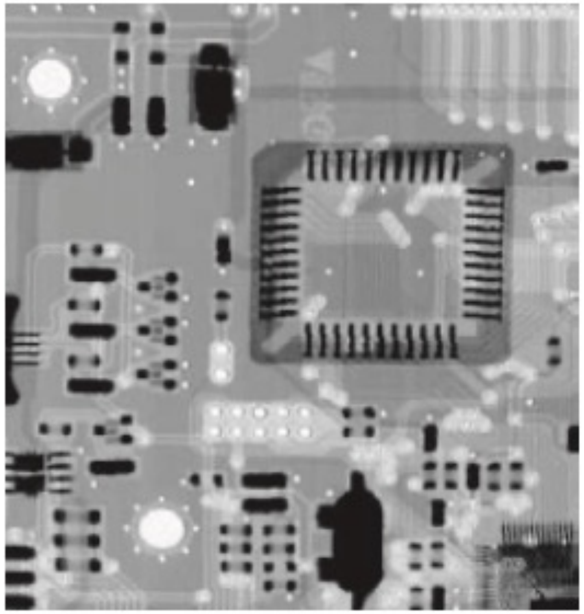


Erosion with a
flat disk SE with
radius of 2 pixels

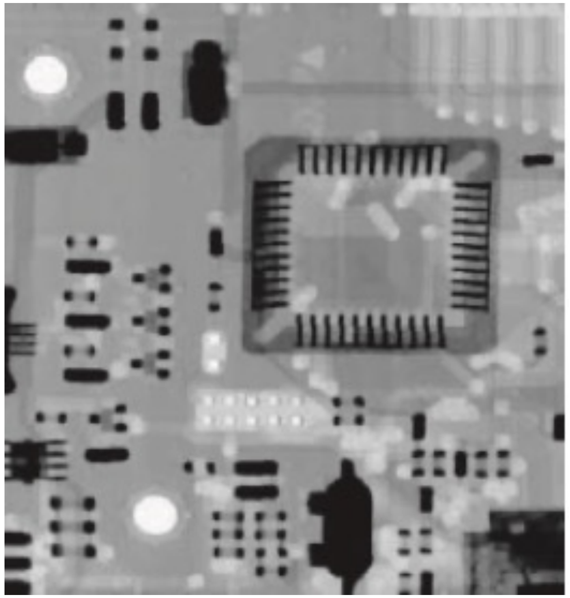


Dilation using the
same SE

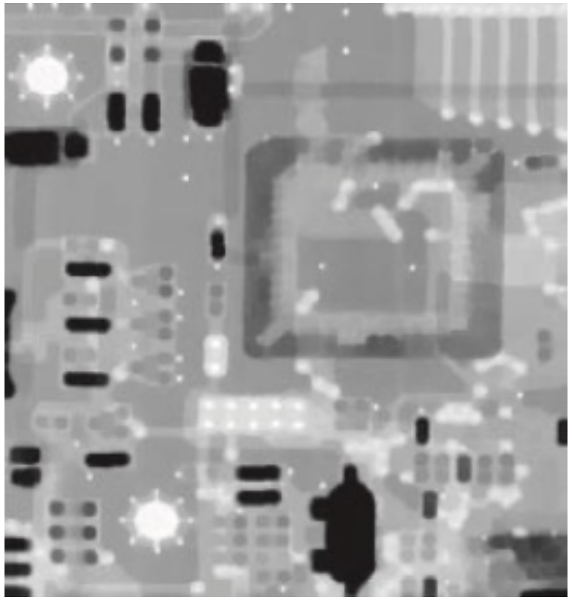
Example of Grayscale Opening and Closing



448 x 425
X-ray image of a PCB



Opening using a disk SE with radius of 3 pixels



Closing using a disk SE with radius of 5 pixels

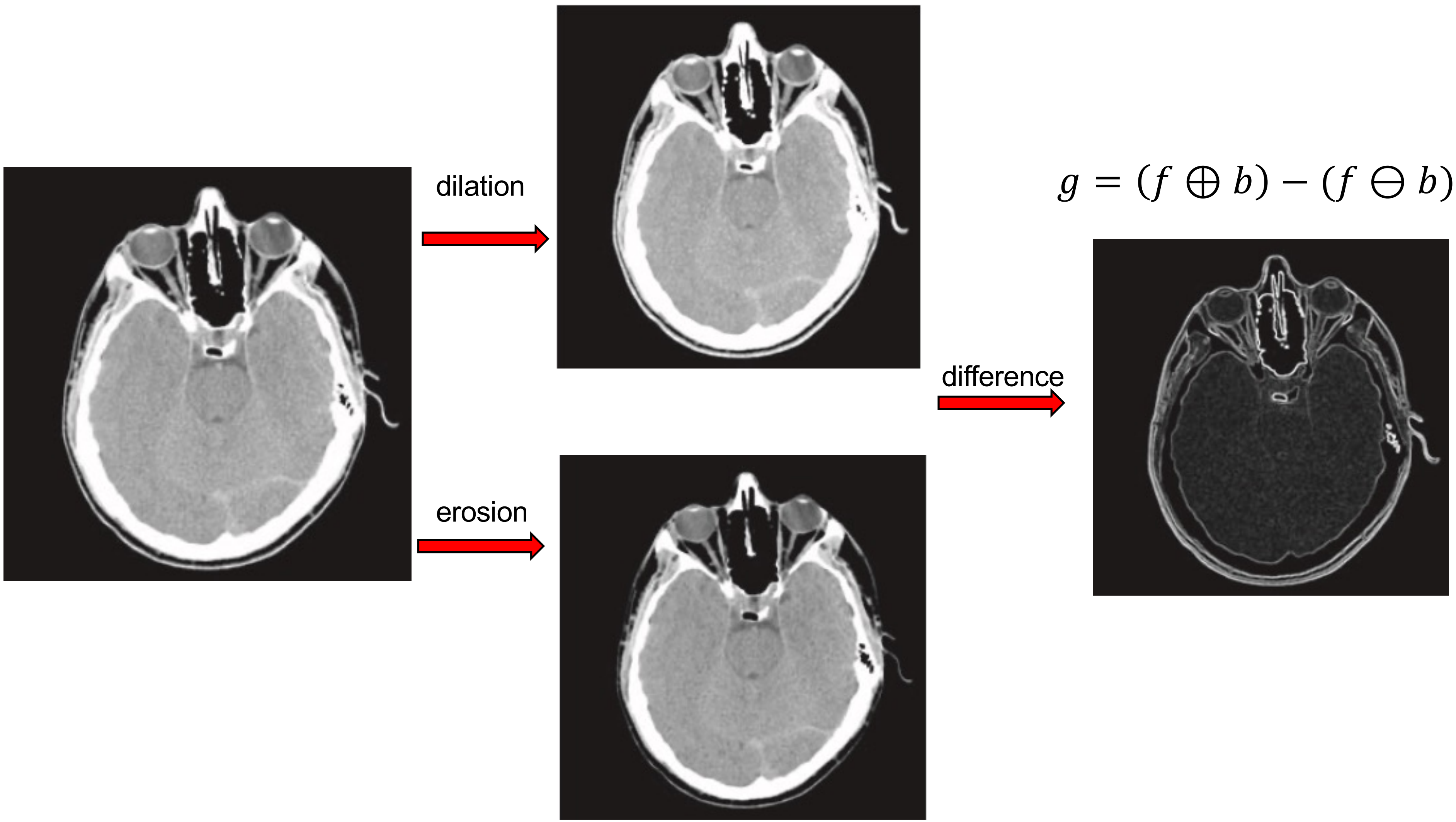
Morphological Gradient

- By subtracting an eroded image from a dilated image, we can obtain the morphological gradient:

$$g = (f \oplus b) - (f \ominus b)$$

- Let us apply basic operation of dilation, erosion, opening and closing tp grayscale instead of just binary images.
- For this discussion, $f(x, y)$ is a grayscale image and $b(x, y)$ is a structuring element. These are functions that assign an intensity value to each distinct pair of coordinate (x, y).
- The structuring elements may take various forms:

Example of using Morphological Gradient



$$g = (f \oplus b) - (f \ominus b)$$

Top-Hat and Bottom-Hat Transformations

- Combining image subtraction with openings and closings results in top-hat and bottom-hat transformations.
- Top-hat transformation is:

$$T_{hat}(f) = f - (f \circ b) = f - (f \ominus b) \oplus b$$

- Bottom-hat transform is:

$$T_{bot}(f) = (f \cdot b) - f = (f \oplus b) \ominus b - f$$

Example of Top-Hat and Bottom-Hat Transformations